Fair Judgement?

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Contents

1 Introduction

In 1989, National Revenue Service of Canada (NRSC) offered a case to a Canadian court that,in a test for promotion,lower pass rates among women than among men were explained by differences in rates of college attendance. The court accepted this explanation. The NRSC announced that the rates of college attendance is a variable that was not directly observed by the court.

1.1 Story

In the case of NRSC, the Revenue Service held a psychological test, the General Intelligence Test, for promoting employees to the position of collections enforcement clerk. As a result of this test, 59% of males and 27% of females passed (see Table 1). Under Canadian law, the Revenue Service must provide an evidence that the test is a reliable and efficient way for selecting candidates according to their merit. The Revenue Service showed that 52% of males and 25% of females had some college education (see Table 2). They defended by referring this fact. The Appeals Board accepted the Revenue Service's claim that there was no obviously difference between male and female pass rates because it merely reflected a difference in cognitive ability that was also evident in the data on college education. Hence, the Appeals Board concluded that the proportions of passing were simply in line with the proportions of college education even though the Revenue Service did not offer pass rate data by sex and education into evidence. Here, we want to ask the following questions.

1.2 Questions:

- Was the Board's judgement in error given the evidence it chose to examine?
- Is the difference in passing rates for males and females consistent with the difference in college attendance rates?
- Would males and females who were the same in term of college attendance have similar passing rates?
- Is the difference in passing rates too large to merely reflect a difference in college attendance?

Table 1: Frequency of Passing

	Passed	Failed	Total	Percentage Passed
Female	68	183	251	27%
Male	68	47	115	59%
Total	136	230	366	37%

Table 2: Frequency of College Attendance

	Some College	No College	Total	Percentage Collegge
Female	63	188	251	25%
Male	60	55	115	52%
Total	123	243	366	34%

2 Methods and Results

2.1 Comparing Proportions in Table 1

- **Difference of Proportions**

  We treat the two rows in Table 1 as independent binomial samples. Of the N_1=251 female, 68 passed the test over the course of the study, a proportion of p_1=68/251=0.27. Of the N_2=115 male, 68 passed, a proportion of p_2=0.59. The sample difference of proportions is 0.27-0.59=-0.32. This difference has an estimated standard error of

  $$\sqrt{\frac{(0.27)(0.73)}{251} + \frac{(0.59)(0.41)}{115}} = 0.054$$

  A 95% confidence interval for the true difference $\pi_1 - \pi_2$ is -0.32$\pm$ 1.96(0.054), or(-0.43, -0.21). Since this interval contains only negative values, we conclude that $\pi_1 - \pi_2 < 0$; that is, $\pi_1 < \pi_2$, so female appears to have lower pass rate.

- **Relative Risk**

  Two groups with sample proportions of p_1 and p_2 have a sample relative risk of $\frac{p_1}{p_2}$. For Table 1, the sample relative risk is $p_1/p_2 = 0.27/0.59 = 0.45$. The sample proportion of pass cases was 122% higher for male. Using computer software (SAS), we find that a 95% confident that, the proportion of pass cases for female is between 0.356 and 0.590 times the proportion of pass cases for male.

- **The Odds Ratio**

  For female, the estimated odds of passed equal 68/183=0.37. The value 0.37 means there were 37 "passed" responses for every 100 "failed" response. The estimated odds equal 68/47=1.45 for male, or 145 "passed" response per every 100 "failed" responses.

The sample odds ratio equals $\hat{\theta} = 0.37/1.45 = 0.255$. The estimated odds of test for female equal 0.255 times the estimated odds for male. The estimated odds were 292% higher for male.

- **Chi-Squared Tests of Independence**
  In two-way contingency tables, the null hypothesis of statistical independence of two response has the form
  $$H_0 : \pi_{ij} = \pi_{i+}\pi_{+j}$$
  for all i and j. The marginal probabilities then specify the joint probabilities. The Pearson chi-squared test statistics are X^2=34.669, and Likelihood-ratio statistics G^2=34.149, based on df=1. The reference chi-squared distribution has a mean of df=1 and a standard deviation of $\sqrt{2df} = 1.41$, so a value of 34.0 is fairly for out in the right-hand tail. Each statistic has a p-value of 0.001. This evidence of association would be rather unusual if the variables were truly independent. Both test statistics suggest that passed or failed the test and gender are associated.

2.2 Comparing Proportions in Table 2

- **Difference of Proportions**
  We treat the two rows in Table 2 as independent binomial samples. Of the N_1=251 female, 63 some college over the course of the study, a proportion of p_1=63/251=0.25. Of the N_2=115 male, 60 some college, a proportion of p_2=0.52. The sample difference of proportions is 0.25-0.52=-0.27. This difference has an estimated standard error of
  $$\sqrt{\frac{(0.25)(0.75)}{251} + \frac{(0.52)(0.48)}{115}} = 0.054$$
  A 95% confidence interval for the true difference $\pi_1 - \pi_2$ is -0.27$\pm$ 1.96(0.054), or(-0.38, -0.16). Since this interval contains only negative values, we conclude that $\pi_1 - \pi_2 < 0$; that is, $\pi_1 < \pi_2$, so female appears to have lower college attendance rate.

- **Relative Risk**
  Two groups with sample proportions of p_1 and p_2 have a sample relative risk of $\frac{p_1}{p_2}$. For Table 1, the sample relative risk is $p_1/p_2 = 0.25/0.52 = 0.48$. The sample proportion of pass cases was 108% higher for male. Using computer software, we

find that a 95% confident that, the proportion of "some college" cases for female is between 0.365 and 0.634 times the proportion of "no college" cases for male.

- **The Odds Ratio**

  For female, the estimated odds of passed equal 63/188=0.36. The value 0.36 means there were 36 "some college" responses for every 100 "no college" response. The estimated odds equal 60/55=1.09 for male, or 105 "some college" response per every 100 "no college" responses.

  The sample odds ratio equals $\hat{\theta} = 0.36/1.05 = 0.307$. The estimated odds of test for female equal 0.343 times the estimated odds for male. The estimated odds were 191% higher for male.

- **Chi-Squared Tests of Independence**

  The Pearson chi-squared test statistics are X^2=25.909, and Likelihood-ratio statistics G^2=25.256, based on df=1. The reference chi-squared distribution has a mean of df=1 and a standard deviation of $\sqrt{2df} = 1.41$, so a value of 25.0 is fairly for out in the right-hand tail. Each statistic has a p-value of 0.001. This evidence of association would be rather unusual if the variables were truly independent. Both test statistics suggest that college attendance and gender are associated.

2.3 Three-Way Contingency Table

There are 3072 possible $2 \times 2 \times 2$ tables of gender by college attendance by test result. According to the report of Gastwirth et al., they provided a possible table with the greatest downward confounding of the crude female-male difference in proportions passing the test and the highest two-sided p-value for the Mantel-Haenszel Test statistic (see Table 3). We use this three-way table to analyze. We regard the test as response variable Y, gender as explanatory variable (X), and college attendance as a single control variable (Z)(see Table 3). We study the effect of gender on the test pass rate, treating college attendance as a control variable.

- **Partial Association**

  We use Table 3 to describe conditional associations between gender and pass rate of the test, controlling for college attendance. When the people were with some college, the pass rate was imposed 11.1% more often for male than for female.

Table 3: $2 \times 2 \times 2$ contingency table

College Attendance	Gender	Passed	Failed	Total	Percentage Passed
Some College	Female	56	7	63	88.9%
	Male	60	0	60	100%
No College	Female	12	176	188	6.4%
	Male	8	47	55	14.5%
Total	Female	68	183	251	27.1%
	Male	68	47	115	59.1%

When people were with no college, the pass rate was imposed 8.1% more often for male than for female. Thus, controlling for college attendance by keeping it fixed, the percentage of "pass" was higher for male than for female.

The bottom portion of Table 3 displays the marginal table for gender and test. Ignoring college attendance, the percentage of "pass" of the test was lower for female than for male.

From Table 3, the estimated odds ratio in the first partial table for which people are with some college, equals

$$\hat{\theta}_{XY(1)} = \frac{56 \times 0}{60 \times 7} = 0$$

, since male with some college all pass the test. In the second partial table, $\hat{\theta}_{XY(2)} = \frac{12 \times 47}{8 \times 176} = 0.40$. The sample odds for female passing the test were 40% of the sample odds for male.

The estimated of the marginal odds ratio equal 0.257.

- **Cochran-Mantel-Haenszel Test**

  The CMH test is a test of null hypothesis that X and Y are conditionally independent, given Z. From Table 3, the CMH= 9.702 with df=1. There is strong evidence against conditional independent (p-value¡0.002). This test is inappropriate when the association varies dramatically among the partial tables. It works best when the X-Y association is similar in each partial table.

- **Testing Homogeneity of Odds Ratio**

There is homogeneous X-Y association in a $2 \times 2 \times 2$ table when

$$\theta_{XY(1)} = \theta_{XY(2)} = \cdots = \theta_{XY(k)}$$

The effect of X on Y is the same at each level of Z and a single number describes the X-Y conditional associations. Conditional independence of X and Y is the special case in which each conditional odds ratio equals 1.0.The *Breslow-day statistic* can test the hypothesis that odds ratio between X and Y is the same at each level of Z. Software (SAS) reports a Breslow-Day statistic equal to 2.793 based on *df*=1 for which p-value=0.095. This evidence does not contradict the hypothesis of equal odds ratios. We are justified in summarizing the conditional association by a single odds ratio for all partial tables.

3 Discussion

The result of the CMH test shows that gender and "pass" of the test are not conditionally independent, given college attendance. The Breslow-Day statistic shows that there exists a homogeneous association. The effect of gender on pass of the test is the same at each level of college attendance.

So let go back the questions mentioned above, and give some comments:

- The Board's judgement "may be" not in error given the evidence it chose to examine.
- The difference in passing rates for males and females is consistent with the difference in college attendance rates (see Table 1 and 2).
- Males and females who were the same in term of college attendance haven't similar passing rates (see Table 3).
- Is the difference in passing rates too large to merely reflect a difference in college attendance? Not exactly so, maybe some related variables are not observed.